

De Sitter Entropy from a Lower Dimensional Black Hole

César Arias,¹ Rodrigo Aros,² and Nelson Zamorano³

^{1,2}*Departamento de Ciencias Físicas
Facultad de Ciencias Exactas
Universidad Andres Bello
Santiago de Chile.*

³*Departamento de Física
Facultad de Ciencias Físicas y Matemáticas
Universidad de Chile
Santiago de Chile.*

Abstract

An alternative way to obtain the entropy of de Sitter spacetime for dimensions higher or equal than five is presented. We show that de Sitter entropy can be obtained as that of a lower dimensional black hole. This result follows from the existence of a one to one correspondence between de Sitter and a spacetime which contains a black hole localized on a p -brane. Specifically, the entropy of five dimensional de Sitter can be obtained as the entropy of a BTZ black hole localized on a 2-brane. Therefore, the microstates giving rise to such entropy, are those carried by a two dimensional conformal field theory. For dimensions higher than five, de Sitter entropy is matched to the entropy of a Schwarzschild-de Sitter black hole localized on a $(d-3)$ -brane.

¹ce.arias@uandresbello.edu

²raros@unab.cl

³nzamora@ing.uchile.cl

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1 Introduction

The latest observational evidence indicates that the universe is expanding in an accelerated way [1]. This means that at some moment in the future our universe will be described by an asymptotically de Sitter space. A d -dimensional de Sitter space-time (dS_d) is the maximally symmetric (Lorentzian) Einstein manifold with positive curvature, characterized by a positive cosmological constant $\Lambda > 0$. Because it describes an accelerated universe, an observer in the Sitter space does not have access to the whole space and, indeed, can only see part of it. This gives rise to a *cosmological horizon*: any light ray beyond it can never reach this observer. Consistently, the geometry of dS_d exhibits compact spacelike slices, which implies two important facts. Unlike its anti-de Sitter counterpart, the notion of spatial infinity becomes ill defined on dS_d . Moreover, the physical states of any consistent quantum theory of gravity on dS_d must be defined on a finite dimensional Hilbert space [2, 3].

In 1977, Gibbons and Hawking [4] showed that the dS_d cosmological horizon displays the same thermodynamical features that the event horizon of a black hole. Remarkably, its entropy matches exactly the Bekenstein-Hawking prescription [5, 6]. Despite this similitude, the dS_d horizon is observer-dependent and therefore is not clear what are the fundamental properties shared by both types of horizons. We do

not know why both entropies are exactly a quarter of the area (in Planck units) of the respective horizon.

Lately, it has been possible to compute from first principles, the entropy of certain kind of specific black holes [9, 10, 11]. Holography [7, 8] suggests that the microscopic degrees of freedom that result in such entropy should be accounted on or near the horizon and that the entropy should be understood in the usual statistical way, as the logarithm of the microstates of the black hole.

However, similar procedures has not been possible to apply in dS_d , with the exception of the (2+1)-dimensional case [12]. One of the main obstacles to implement previous calculations from black holes into dS_d , together with the lack of spatial infinity notion, is that dS_d does not admit supersymmetric extension [13]. Furthermore, $dS_d \times \mathcal{M}$ (with \mathcal{M} some arbitrary manifold) is not a background for ten, neither eleven dimensional supergravity [14].

In this work we address the problem of the dS_d entropy for dimensions higher or equal than five. We describe a procedure that yields the dS_d entropy from that of a lower dimensional black hole. Our prescription relies on three facts: (1) de Sitter space can be put into a one to one correspondence with another spacetime, that we refer to as the *image space*, and we denote as \widetilde{dS}_d , (2) the image space contains a gravitational source. Specifically, a conical geometry can be induced at the origin of \widetilde{dS}_d , acting as δ -type contribution to the energy-momentum tensor of the system. This source represents a $(d-2)$ -dimensional hypersurface whose induced metric describes a black hole. Therefore, the configuration of the image space can be seen as a codimension-two brane black hole, and (3) under the map $dS_d \rightarrow \widetilde{dS}_d$, the cosmological horizon is mapped into the surface defined by the brane, allowing the identification of degrees of freedom carried by the dS_d horizon with those carried by the brane black hole. Besides, we show that the dS_d entropy matches exactly the entropy of the localized black hole in the image space provided a fixed angular deficit is introduced along the transverse directions to the brane. That is, one can work out the entropy of de Sitter space by inducing a brane black hole within the image space and computing this entropy, instead of the dS_d one.

The paper is organized into three parts. In section 2, the definition and construction of the image space is carried out, and the correspondence between the two geometries is established. In section 3, it is shown that the image space describes a

brane black hole. The entropy of this latter black hole is matched to that of dS_d in section 4. Finally, some remarks and comments are included. We will adopt units such that $\hbar = c = 1$, but keeping Newton's constant $G \neq 1$ throughout.

2 The Correspondence

In this section, we proceed to construct the image space. Let us consider, as starting point, the d -dimensional de Sitter spacetime (dS_d) in static coordinates

$$ds^2 = - \left(1 - \frac{R^2}{L^2}\right) dT^2 + \frac{dR^2}{\left(1 - \frac{R^2}{L^2}\right)} + R^2 d\Omega_{d-2}^2 \quad (1)$$

where $d\Omega_N^2$ denotes the unitary N -sphere line element and L the dS_d radius, related with the d -dimensional cosmological constant $\Lambda > 0$, through $L^2 = \frac{(d-1)(d-2)}{2\Lambda}$. The above set of coordinates cover only part of the space, as known, the patch defined by $0 \leq R \leq L$, which represents an observer located at $R = 0$ surrounded by a cosmological horizon at $R = L$. Each spacelike surface at $R = R_0 = \text{constant}$, on the other hand, has the topology of a $(d-2)$ -sphere of radius R_0 .

Having the previous configuration in mind, let us introduce the following set of transformations : first, a coordinate change $R = R(z)$ such that the origin $R = 0$, be mapped into $z \rightarrow \infty$ and the cosmological horizon L , to $z = 0$. In this way, the new coordinate will cover the whole interval $0 \leq z < \infty$. Explicitly,

$$R = \frac{L}{\cosh(z/L)}. \quad (2)$$

Secondly, still in (1), replace the horizon, defined by the spacelike (hyper)surface $R = L$, by a $(d-2)$ -dimensional spacetime, namely¹ $d\hat{s}_{d-2}^2$

$$\begin{aligned} L^2 d\Omega_{d-2}^2 \rightarrow d\hat{s}^2 &= \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu \\ &= -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-4}^2 \end{aligned} \quad (3)$$

¹About the notation : here and in what follows, the *hat* will denote objects intrinsically $(d-2)$ -dimensional, and therefore, the quantities with this label will depends only on the x -coordinates, which ensures the Lorentz invariance of the subspace defined by \hat{g} . In this way, Greek indexes run from 0 to $(d-2) - 1$, whilst capital ones, will run over the whole space.

Finally, to preserve the Lorentzian signature of the whole space, a third substitution is in order. For this it will be proposed

$$T \rightarrow iL\phi. \quad (4)$$

After performing the sequence of transformations (2)-(4) into (1) the resulting new geometry is described by

$$ds^2 = \frac{1}{\cosh^2(z/L)} [dz^2 + L^2 \sinh^2(z/L) d\phi^2 + \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu], \quad (5)$$

which is an Einstein manifold provided

$$f(r) = \beta - \frac{\mu}{r^{d-5}} - \frac{r^2}{L^2} =: f_d(r). \quad (6)$$

In this equation β and μ are two parameters depending on the dimension. It can be shown that β is an arbitrary constant for $d = 4, 5$, and it must be fixed to the unit for $d \geq 6$. On the other hand, μ will be related with the black hole structure of the \hat{g} -subspace.

It is important to point out that the modifications displayed by equations (3) and (4) are not diffeomorphisms. Indeed, the spaces (1) and (5) are topologically inequivalent. This can be readily checked by computing some curvature invariants. For instance, while Ricci scalar and the full contraction of the Ricci tensor are the same for both spaces

$$R = \frac{d(d-1)}{L^2} ; \quad R_{AB}R^{AB} = \frac{d(d-1)^2}{L^4}, \quad (7)$$

the Kretschmann scalars differ. In fact for the line element (5)

$$\begin{aligned} K := R_{ABCD}R^{ABCD} &= \frac{2d(d-1)}{L^4} + \mu^2 (d-3)(d-4)^2(d-5) \frac{\cosh^4(z/L)}{r^{2(d-3)}} \\ &=: K_{dS} + \mu^2 \gamma(d) \frac{\cosh^4(z/L)}{r^{2(d-3)}}, \end{aligned} \quad (8)$$

whereas for dS_d space the last term vanishes. In this case, the new space described by (5) is not a maximally symmetric space and furthermore, for $d \geq 6$, it has two curvature singularities at $r = 0$ and $z \rightarrow \infty$ (the image of the dS_d origin under (2)). The first singularity reveals the existence of a *black hole*.

In what follows, the new geometry (5)-(6) is called image space and denoted by \widetilde{dS}_d . The table below summarizes the correspondence between spaces.

dS_d	\widetilde{dS}_d
Coordinates (R, T, Ω_{d-2})	Coordinates (z, ϕ, x^μ)
Origin $R = 0$	$z \rightarrow \infty$
Horizon $R = L$ (spacelike surface $L^2 d\Omega_{d-2}^2$)	$z = 0$ (timelike surface $d\hat{s}^2$)
$K = K_{dS}$	$K = K_{dS} + \mu^2 \gamma(d) \frac{\cosh^4(z/L)}{r^{2(d-3)}}$

3 Brane Localization

In this section, we show that the spacetime \widetilde{dS}_d can be seen as a codimension two brane black hole². More precisely, the $(d-2)$ -dimensional surface defined by $z = 0$ (the brane), can be localized on a conical defect in d dimensions, acting as a gravitational source. The first clue of the existence of this defect, is the asymptotic behavior of the metric around the origin

$$ds^2|_{z \rightarrow 0} \simeq dz^2 + z^2 d\phi^2 + \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu \quad (9)$$

so that if we identify the angular coordinate $\phi \rightarrow \alpha\phi$, with $\alpha \neq 1$, a conical singularity will be induced, characterized by a deficit angle

$$\Delta\phi = 2\pi(1 - \alpha). \quad (10)$$

This angular deficit can be interpreted as the geometrical effect of a gravitational source³, with an energy-momentum (density) proportional to $\Delta\phi$. The argument to conclude this, goes as follows [21]: consider a d -dimensional spacetime characterized by the metric ansatz in normal (Gaussian) coordinates

$$ds^2 = d\rho^2 + B^2(\rho) d\phi^2 + W^2(\rho) \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu \quad (11)$$

then, if the function $B = B(\rho)$ admits, for $\rho \rightarrow 0$, the expansion

$$B(\rho) = \alpha\rho + O(\rho^2) \quad (12)$$

with $\alpha \neq 1$, we have a conical singularity in the (ρ, ϕ) -plane, located at $\rho = 0$. The space in the vicinity of this conical singularity will be regular, that is, any invariant

²For a review of this topic, see [15, 16] and references in it. Previous literature regarding the codimension-two scenario includes [17, 18, 19, 20, 22, 21].

³ This is the codimension-two analogue of considering the discontinuity in the extrinsic curvature, in the codimension-one case, as a δ -source of energy-momentum at the brane position.

constructed from de Riemann curvature tensor remains finite around $\rho \rightarrow 0$, if we demand the following boundary conditions (primes indicates derivatives with respect to ρ)

$$\begin{aligned} B|_{\rho=0} &= 0; \quad B'|_{\rho=0} = \alpha; \\ W^2|_{\rho=0} &= 1; \quad (W^2)'|_{\rho=0} = 0. \end{aligned} \quad (13)$$

Due to equations (12) and (13), a δ -type singularity arises to solve the $(\mu\nu)$ component of the Einstein equations as

$$G_{\mu\nu}|_{\text{sing}} = -\frac{B''}{B} g_{\mu\nu} = (1 - \alpha) \frac{\delta(\rho)}{B} g_{\mu\nu} + (\text{non-singular terms}). \quad (14)$$

In terms of the d dimensional indexes this δ -like singular behavior can be described by an energy-momentum tensor of the form⁴

$$T_{AB} = \delta_A^\mu \delta_B^\nu \hat{T}_{\mu\nu}(x) \frac{\delta(\rho)}{2\pi B}. \quad (15)$$

Matching the singular contributions (14) and (15) from both sides of Einstein's equations at the brane position, $\rho = 0$, it is obtained the general form of the $(d - 2)$ -dimensional density of energy-momentum

$$\hat{T}_{\mu\nu}(x) = \frac{1}{8\pi G_d} [2\pi (1 - \alpha)] \hat{g}_{\mu\nu}(x) = \frac{\Delta\phi}{8\pi G_d} \hat{g}_{\mu\nu}(x), \quad (16)$$

where G_d denotes Newton's constant in d -dimensions. Thus, we see that the brane must have an energy-momentum tensor proportional to their induced metric [20]. It is precisely this fact what allows us to consider a(n) (anti-)de Sitter $(d - 3)$ -brane embedded in d dimensions.

Returning to our case, we apply the above prescription to the image space \widetilde{dS}_d , through the following changes

$$\rho(z) = 2L \tan^{-1} \left[\tanh \left(\frac{z}{2L} \right) \right] ; \quad 0 \leq \rho \leq \frac{\pi L}{2} \quad (17)$$

$$\phi \rightarrow \alpha\phi \quad (18)$$

⁴Again, we can think in the codimension-one analogue of this balance: in that case, in order to capture the effect of the brane as a source, we need to add the Gibbons-Hawking term to the Einstein-Hilbert action, that which makes the variational principle well defined [23].

obtaining

$$ds^2 = d\rho^2 + \alpha^2 L^2 \sin^2(\rho/L) d\phi^2 + \cos^2(\rho/L) \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu. \quad (19)$$

In this coordinates, it is clear that the conditions (12) and (13) are fulfilled, and consequently, we can consider the $(d-2)$ -dimensional submanifold defined by \hat{g} , as a gravitational source (a p -brane). The geometry of this brane, as mentioned before, should be consistent with (16). Hence, we consider a source with an energy-momentum of the form

$$\hat{T}_{\mu\nu}(x) = \sigma \hat{g}_{\mu\nu}(x) \quad (20)$$

where σ defines brane tension, understood as the energy density contribution of the brane embedding. Comparing expressions for the energy-momentum tensor (16) and (20), we obtain the relation between the angular deficit and the internal structure of the brane [24]

$$\sigma = \frac{\Delta\phi}{8\pi G_d}. \quad (21)$$

From the brane point of view, Einstein field equations should be fulfilled. Thus, if we denote by λ_{eff} the effective cosmological constant of the brane, then \hat{g} must satisfy $\hat{G}_{\mu\nu} + \lambda_{\text{eff}} \hat{g}_{\mu\nu} = 0$, where λ_{eff} should takes into account the mean energy density of the empty space (what we usually call cosmological constant λ), and of the brane tension σ , given by (21). That is

$$\lambda_{\text{eff}} = \lambda - 8\pi G_{d-2} \sigma. \quad (22)$$

This can be explicitly calculated by dimensional reduce the Einstein-Hilbert action, using the metric ansatz (11). Starting from d -dimensions, we get

$$\begin{aligned} I[g] &= \frac{1}{16\pi G_d} \int d^d x \sqrt{-g} (R - 2\Lambda) \\ &= \frac{1}{16\pi G_d} \int d\rho d\theta B(\rho) W^{d-4}(\rho) \int d^{d-2} x \sqrt{-\hat{g}} \left[\hat{R} - 2\Lambda \frac{d-4}{d-2} W^2(\rho) \right] \\ &= \frac{1}{16\pi G_d} \left(\frac{2\pi \alpha L^2}{d-3} \right) \int d^{d-2} x \sqrt{-\hat{g}} \left[\hat{R} - 2\Lambda \frac{(d-3)(d-4)}{(d-1)(d-2)} \right]. \end{aligned} \quad (23)$$

From the last line, we identify Newton's constant in $(d-2)$ dimensions as

$$G_{d-2} = \left(\frac{d-3}{2\pi \alpha L^2} \right) G_d. \quad (24)$$

Including now the effect of the gravitational source (20)

$$I[\sigma] = \int d^{d-2}x \sqrt{-\hat{g}} \sigma \quad (25)$$

we obtain the effective $(d-2)$ -dimensional cosmological constant on the brane

$$\lambda_{\text{eff}} = \frac{(d-3)(d-4)}{(d-1)(d-2)} \Lambda - 8\pi G_{d-2} \sigma =: \chi(d) \Lambda - 8\pi G_{d-2} \sigma. \quad (26)$$

The latter expression is in agreement with (22), providing $\lambda = \chi(d) \Lambda$. Moreover, in absence of the source ($\sigma = 0$), the \hat{g} -submanifold has a positive λ_{eff} . However, for a non vanishing tension, we have three possibles values of it. Combining (21) and (24), equation (26) becomes

$$\lambda_{\text{eff}} = \frac{d-3}{2L^2} (d-2-2\alpha^{-1}). \quad (27)$$

Hence the parameter α , responsible for regulating the angular deficit, determines the value of the induced cosmological constant

$$\alpha \begin{cases} > \frac{2}{d-2} & \text{dS brane,} \\ = \frac{2}{d-2} & \text{(Ricci) flat brane,} \\ < \frac{2}{d-2} & \text{AdS brane.} \end{cases} \quad (28)$$

This is an useful result, because it gives us the freedom to localize, depending on the size of the angular deficit, different types of branes at the origin of image space. In the next section, we will use this freedom to map the entropy of dS_d into the entropy of the brane black hole.

4 Entropy

So far we have constructed a new spacetime, which is the image of de Sitter under the transformations (2), (3) and (4). The main feature of this geometry is that can be seen as a black hole localized on the $z = 0$ surface. This surface (the brane), is the image of the cosmological horizon of the original de Sitter space. Having established this correspondence, we conjecture that the entropy of dS_d , is equivalent to the entropy of the localized black hole in the image space.

4.1 Five dimensional case

Here, we focus on the case of five dimensions. The induced metric on the brane is described by the following f -term

$$f_5(r) = \beta - \mu \pm \frac{r^2}{\ell_{\text{eff}}^2} \quad (29)$$

where the sign of the last term depends on the domain in which α is defined (28). We will show that the image space \widetilde{dS}_5 contains a BTZ black hole [25] localized on a 2-brane⁵. This brane is in correspondence with the dS_5 cosmological horizon, and therefore we identify the dS_5 horizon degrees of freedom with those that result in the BTZ entropy.

In order to carry this procedure out, and according to (28), we need to impose the AdS bound $\alpha < \frac{2}{3}$. Consequently, the effective cosmological constant will be related with AdS_3 effective radius ℓ_{eff} in the usual way: $\lambda_{\text{eff}} = -1/\ell_{\text{eff}}^2$. Then, equation (27) gives

$$\ell_{\text{eff}}^2 = \frac{\alpha L^2}{2 - 3\alpha}. \quad (30)$$

Note that for $\alpha = 1$ the previous relation is consistent with (6). Now in (29), fixing μ and β in terms of ℓ_{eff} and some characteristic length r_H , in the following way

$$\beta - \mu = -\frac{r_H^2}{\ell_{\text{eff}}^2} \quad (31)$$

the metric for the 2-brane will be

$$d\hat{s}^2 = -\left(\frac{r^2 - r_H^2}{\ell_{\text{eff}}^2}\right) dt^2 + \left(\frac{r^2 - r_H^2}{\ell_{\text{eff}}^2}\right)^{-1} dr^2 + r^2 d\phi^2 =: ds_{BTZ}^2 \quad (32)$$

This is the line element for the (non rotating) BTZ black hole, with event horizon located at $r = r_H$. It characterizes the geometry of the 2-brane, which acts as a gravitational source embedded within the image space \widetilde{dS}_5 .

Once the embedding is done, the transformation (3) becomes clear: it changes the isometry group of the 3-sphere of radius L (the spacelike surface at $R = L$ in dS_5), by the AdS_3 isometry group. Next, it takes the quotient such that the resulting group

⁵The construction of a dS 2-brane as a surface of dS_5 is possible, too. However, we focus in the study of an AdS 2-brane for reasons that will become clear below.

submanifold is BTZ black hole⁶. Schematically, the change $L^2 d\Omega_{d-2}^2 \rightarrow d\hat{s}^2$ defined in (3), in terms of the respective isometry groups, has the form

$$SO(4) \rightarrow SO(2,2)/\Sigma \cong \text{BTZ} \quad (33)$$

where Σ denotes a discrete subgroup of $SO(2,2)$ [26, 27]. Moreover, after to apply the previous transformation, and doing the change $\rho = L\theta$ in the metric (19), the image space line element takes the form

$$ds^2 = L^2(d\theta^2 + \alpha^2 \sin^2 \theta d\phi^2) + \cos^2 \theta ds_{BTZ}^2 \quad (34)$$

with $0 \leq \theta \leq \pi/2$. From here we read off that \widetilde{dS}_d has the topology of a (half of a) 2-sphere (up to deficit angle) times a warped BTZ black hole.

Having constructed the image space, let us compute the entropy at the brane position. Here, the only degrees of freedom that contribute to such entropy, are those from the BTZ event horizon r_H . Then, the entropy on the brane will be given by the Bekenstein-Hawking entropy of the BTZ black hole, that is

$$S_{BTZ} = \frac{2\pi r_H}{4G_3}. \quad (35)$$

As argued in section 3, the only physical effect of fixing the parameter α is the sign that the induced cosmological constant λ_{eff} can take. At this point, there is no loss of generality if we set α to any value consistent with the AdS bound. In particular we can think in⁷

$$\alpha = r_H^{-1} L. \quad (36)$$

Replacing r_H from the above choice into the BTZ entropy (35), and using (24) to

⁶It is a well known fact that BTZ black hole is obtained by discrete identifications of AdS_3 . In terms of the isometry groups, it is given by the quotient $SO(2,2)/\Sigma$, with Σ a discrete subgroup of $SO(2,2)$. Further details of this construction can be reviewed in [26, 27].

⁷ Let us analyze the freedom we have for this choice: for the non rotating version of BTZ black hole the event horizon depends on its mass and of the AdS_3 radius, in our case, $r_H = \sqrt{M} \ell_{\text{eff}}$. Consequently, and according with the relation (30), setting $\alpha = r_H^{-1} L$, is equivalent to pick up one of the roots of the cubic equation $M\alpha^3 + 3\alpha - 2 = 0$, such that $\alpha < 2/3$. This is always possible. The previous reasoning makes clear, too, why this choice is not consistent with the localization of a de Sitter 2-brane. If so, we would need, by one hand to take the bound $\alpha > 2/3$, but on the other hand, in order to have a positive mass $M > 0$, we would need $\alpha < 0$, which is a contradiction.

rewrite G_3 in terms of G_5 , we get⁸

$$S_{BTZ} = \frac{1}{4G_5} \left[\frac{2\pi^2}{\Gamma(2)} \right] \alpha r_H L^2 = \frac{S_3 L^3}{4G_5} = \frac{A_{dS5}}{4G_5} \quad (37)$$

where A_{dS5} denotes the area of the dS_5 cosmological horizon. Thereupon, the previous equation suggests the equivalence (before conjectured) between the entropy of the full five dimensional de Sitter space and that of the BTZ black hole localized on the 2-brane

$$S_{dS5} = S_{BTZ} \quad (38)$$

This result gives insights about what are the possible microstates that the entropy of dS_5 is counting. As pointed out by [11], given the fact that any gravity theory on AdS_3 is a conformal field theory at the boundary [28, 29], the entropy of the BTZ black hole can be computed through the Cardy's formula [30], counting the asymptotic growth of the number of the states of the CFT_2 . Then, equation (38) suggests that the entropy of dS_5 is indeed related with the density of states of the Hilbert space on which the CFT_2 is defined.

4.2 Higher dimensional case

Finally, let us consider the case in which the dimension of the spacetime is higher or equal than six. Depending on the α -bound (28), the brane geometry is that of a Schwarzschild black hole for a null λ_{eff} , or a Schwarzschild-(anti) de Sitter ($S(A)dS_{d-2}$) black hole for a non zero λ_{eff} . However, as in the case of five dimensions, there is a restriction which determines the nature of the brane. For this reason we focus on the embedding of a SdS_{d-2} black hole. That is, we consider $\alpha > \frac{2}{d-2}$.

At the brane location, the only contribution to the entropy is that coming from the SdS_{d-2} black hole. Denoting by r_H the event horizon of the black hole, we have

$$S_{BH} = \frac{A}{4G_{d-2}} = \frac{S_{d-2} r_H^{d-4}}{4G_{d-2}}. \quad (39)$$

⁸The N -volume of the unitary N -sphere is given by

$$S_N = \frac{2\pi^{\frac{N+1}{2}}}{\Gamma(\frac{N+1}{2})}$$

Using again the relation between Newton's constants (24), the previous equation takes the form

$$S_{BH} = \left(\frac{2\pi \alpha L^2}{d-3} \right) \frac{S_{d-4} r_H^{d-4}}{4 G_d}. \quad (40)$$

Now, because the SdS_{d-2} black hole horizon is always located at $r_H < \ell_{\text{eff}}$, necessarily $r_H = \gamma \ell_{\text{eff}}$, for some $\gamma < 1$. Therefore, we can take⁹

$$\gamma = \alpha^{\frac{1}{4-d}} \left(\frac{L}{\ell_{\text{eff}}} \right) < 1. \quad (41)$$

Under this choice, the Bekenstein-Hawking entropy of the brane black hole becomes

$$S_{BH} = \left[\frac{2\pi S_{d-4}}{d-3} \right] \frac{L^{d-2}}{4 G_d} = \left[\frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})} \right] \frac{L^{d-2}}{4 G_d} = \frac{S_{d-2} L^{d-2}}{4 G_d} = \frac{A_{dS}}{4 G_d} \quad (42)$$

which is exactly the entropy of the full dS_d spacetime. Then, we claim that the entropy of the dS_d can be understood as the entropy of the localized SdS_{d-2} black hole in the image space

$$S_{dS} = S_{BH}. \quad (43)$$

5 Concluding Remarks

In this paper we have proved that de Sitter entropy for dimensions higher or equal than five, can be obtained by computing the entropy of a lower dimensional black hole. To conclude this, we constructed a map from dS_d into a new spacetime, \widetilde{dS}_d . The geometry of \widetilde{dS}_d describes a timelike submanifold whose induced metric has a codimension-two brane black hole structure. Under $dS_d \cong \widetilde{dS}_d$, the preimage of the brane is precisely the cosmological horizon of de Sitter space, allowing the identification of the microstates responsible for de Sitter and black hole entropies, respectively.

Summarizing, the dS_d entropy can be computed by inducing a brane black hole in the image space, and calculating the Bekenstein-Hawking entropy of this latter. Using this approach we analyzed two cases, five and higher dimensions.

⁹This choice is equivalent to fix α . Here, the need for a de Sitter brane becomes evident. It follows from the relation between ℓ_{eff} and L , that demanding $\gamma < 1$, is equivalent to require $|d-2-2\alpha^{-1}| < (d-4)\alpha^{\frac{2}{d-4}}$. The above inequality is only fulfilled if $\alpha > \frac{2}{d-2}$, which is the bound for a dS brane.

In the five dimensional case, the previous correspondence takes a particularly interesting form. The spatial boundary of dS_5 (its horizon) is mapped into the AdS_3 slice immerse in the image space (the AdS 2-brane, which after identifications becomes the BTZ black hole). From this, it was shown that $S_{dS_5} = S_{BTZ}$. But at the same time AdS_3 is dual to a two dimensional conformal field theory [28, 29]. The sequence $dS_5/AdS_3/CFT_2$ suggests that the entropy of dS_5 can be understood in terms of the microstates of the CFT_2 . Therefore, its microstates can be counted in the same way that those of the BTZ black hole [11], that is, as the asymptotic density of states of the Hilbert space on which the CFT_2 is defined, through the Cardy's formula [30].

For dimensions higher than five, the situation is different. First, because the image space posses two curvature singularities, as we can read from the Kretschmann scalar (8). Both singularities, which are the image of the origin and the cosmological horizon of dS_d , exist simultaneuosly. Second, because in order to match the entropies from both sides of $dS_d \cong \widetilde{dS}_d$, we need to induce a positive cosmological constant on the brane, fixing the value of the parameter α as indicated in (28) for the $\lambda_{\text{eff}} > 0$. The resulting geometry of the image space is then a $(d - 2)$ -dimensional Schwarzschild-de Sitter brane black hole. Identifying the degrees of freedom of the SdS_{d-2} black hole horizon with those of the dS_d cosmological horizon, it was shown the equivalence $S_{dS} = S_{BH}$.

The previous results seem to indicate that the problem of entropy for de Sitter and black hole spaces, different in principle, are indeed related by a duality.

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